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Correlation Functions in the Itzykson-Zuber Model

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Abstract. The n-point function for the integral over unitary matrices with Itzykson-Zuber measure is reduced to the integral over the Gelfand-Tzetlin table; the integrand (for generic n) is given by linear exponential times the rational function. For $n = 2$ and in some cases for $n > 2$ later in fast is the polynomial and this allows to give an explicit and simple expression for all 2-point and a set of n-point functions. For the most general n-point function a simple linear differential equation is constructed.

1. Introduction

In this letter I'll consider the following correlation function:

$$\langle g_{i_1 j_1} g^+_{k_1 l_1} \cdots g_{i_n j_n} g^+_{k_n l_n} \rangle = \int [d\mu(g)] \exp[\mathrm{Tr}(gMg^+N)]\, g_{i_1 j_1} g^+_{k_1 j_1} \cdots g_{i_n j_n} g^+_{k_n l_n} \,. \tag{1.1}$$

Here g is the N dimensional unitary matrix and M and N are Hermitian. The measure of integration is Haar measure. Without lack of generality we could assume that M and N are diagonal.

For the case of $n = 0$ (partition function) this integral was calculated by Harish-Chandra [1] and Itzykson and Zuber [2] a long time ago. Here we will use the method previously used in a similar problem in [3]; this simple algebraic method is known in the literature on representation theory since 1950 [4]. Let me mention that the main motivation to look on the integral (1.1) is related to investigation of the Kazakov-Migdal model [5]; also, this kind of integrals might be interesting for string theory related matrix models [6].

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