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Vacuum Expectation Values of Products of Chiral Currents in 3 + 1 Dimensions

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Abstract. An algebraic rule is presented for computing expectation values of products of local nonabelian charge operators for fermions coupled to an external vector potential in $3+1$ space-time dimensions. The vacuum expectation value of a product of four operators is closely related to a cyclic cocycle in noncommutative geometry of Alain Connes. The relevant representation of the current is constructed using Kirillov's method of coadjoint orbits.

1. Introduction

In $1+1$ space-time dimensions it is known that a normal ordering of local charge operators is sufficient to make them well-defined in a suitable dense domain of a fermionic Fock space. Assuming that the physical space is compactified as the circle S^1, the normal ordered charge densities define a representation of a central extension $\widehat{LG}$ of the (Lie algebra of) the loop group LG corresponding to a compact gauge group G. In the case of chiral fermions the central term is nontrivial and gives rise to a highest weight representation of $\widehat{LG}$. The Lie algebra of the group is an affine Kac-Moody algebra.

In higher dimensions even the normal ordered current densities do not give well-defined operators in the Fock space. Even the state created from the vacuum by an action of a typical element of the current algebra has an infinite norm. This reflects the fact that the automorphisms of the algebra of canonical anticommutation relations (CAR) generated by gauge transformations are not implementable by unitary transformations in the Fock space in space dimensions higher than one. (For a thorough discussion of CAR representations see [A].) In other words, a gauge transformation tends to take a state in the Fock space to a vector in a different Fock space corresponding to an *inequivalent* representation of CAR.

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