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# Existence of Black Hole Solutions for the Einstein-Yang/Mills Equations

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**Abstract.** This paper provides a rigorous proof of the existence of an infinite number of black hole solutions to the Einstein-Yang/Mills equations with gauge group $SU(2)$, for any event horizon. It is also demonstrated that the ADM mass of each solutions is finite, and that the corresponding Einstein metric tends to the associated Schwarzschild metric at a rate $1/r^2$, as $r$ tends to infinity.

## 1. Introduction

In this paper we prove that the Einstein-Yang/Mills (EYM) equations, with $SU(2)$ gauge group, admit an infinite family of "black-hole" solutions having a regular event horizon, for every choice of the radius $r_H$ of the event horizon. The solutions obtained are indexed by a "winding number". Moreover, we prove that the ADM mass, [2] of each solution is finite, and the corresponding Einstein metric tends to the associated Schwarzschild solution in the far field. Some of our results were observed numerically in [4, 5]; see also [3]. Numerical discussions of the stability properties of some of these solutions can be found in [6, 9].

The existence problem reduces to finding solutions of the following system of ordinary differential equations in the region $r \geq r_H$:

$$\begin{aligned} r^2 A w'' + \left[ r(1-A) - \frac{(1-w^2)^2}{r} \right] w' + w(1-w^2) = 0\,, \\ rA' + (1+2w'^2)A = 1 - \frac{(1-w^2)^2}{r^2}\,, \end{aligned} \tag{1.1}$$

subject to certain boundary conditions. These equations were studied in [7, 8], where the existence of globally defined regular (i.e., non-black-hole) solutions was proved.

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