Commun. Math. Phys. 154, 347–376 (1993)

Communications in
Mathematical
Physics

Orthogonality and Completeness of the Bethe Ansatz Eigenstates of the Nonlinear Schroedinger Model

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Received July 20, 1992; in revised form October 26, 1992

Abstract. A rigorous proof is given of the orthogonality and the completeness of the Bethe Ansatz eigenstates of the N-body Hamiltonian of the nonlinear Schroedinger model on a finite interval. The completeness proof is based on ideas of C.N. Yang and C.P. Yang, but their continuity argument at infinite coupling is replaced by operator monotonicity at zero coupling. The orthogonality proof uses the algebraic Bethe Ansatz method or inverse scattering method applied to a lattice approximation introduced by Izergin and Korepin. The latter model is defined in terms of monodromy matrices without writing down an explicit Hamiltonian. It is shown that the eigenfunctions of the transfer matrices for this model converge to the Bethe Ansatz eigenstates of the nonlinear Schroedinger model.

1. Introduction

The nonlinear Schroedinger model was introduced by Lieb and Liniger [26] in 1963 as the first model of a boson gas that contains a nontrivial adjustable parameter and which can be fully analysed without making approximations. Earlier, Girardeau [20] had introduced a simpler model of a gas of impenetrable bosons in one dimension, but this model behaves effectively as a very high-density gas. Also, the latter model does not have a nontrivial parameter. In many other respects, however, the two models are quite similar and, indeed, Girardeau's model can be obtained from the nonlinear Schroedinger model in the infinite-coupling limit.

In [26], Lieb and Liniger obtained the eigenfunctions of the nonlinear Schroedinger model Hamiltonian with periodic boundary conditions using an "Ansatz" similar to the one used by Bethe in his analysis [11] of the one-dimensional Heisenberg model. They did not prove. however, that the set of eigenfunctions thus obtained is actually complete. Many other models have since been shown to be soluble by means of the Bethe Ansatz method and generalisations thereof. (See for example [19, 6, 18, 34 and 14].) In particular Baxter's solution of the anisotropic Heisenberg chain [5] was a major breakthrough. Another