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The Quotient Construction for a Class of Compact Einstein Manifolds

Brett McInnes

Department of Mathematics, National University of Singapore, 10 Kent Ridge Crescent, 0511 Republic of Singapore

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Abstract. Given any Einstein manifold M^E, one can obtain further examples of Einstein manifolds by taking the quotient M^E/G by a freely acting, properly discontinuous group of isometries. We study this method in the case in which M^E is Kählerian, M^E/G is compact, and the Ricci curvature is non-negative. In many cases, the candidates for G can be completely classified.

1. Introduction

The Einstein manifolds constitute perhaps the most interesting special class of Riemannian manifolds, and their properties have frequently attracted the attention of physicists: one thinks of the application of Myers' theorem by Freund and Rubin [5], of Yau's theorem by Candelas et al. [3], of hyperkähler geometry by workers in supersymmetric sigma models [8], and so on. A more complete understanding of the full range of Einstein manifolds would clearly be highly desirable both in physics and in mathematics [2].

Given any Einstein manifold M^E, one has a canonical procedure for constructing further examples of the same dimension. If a group G acts isometrically, freely, and properly discontinuously on M^E, then M^E/G is also an Einstein manifold. The distinction between M^E and M^E/G is purely global, but this global distinction can have physical consequences: in string theory, the fact that Calabi–Yau manifolds of the form M^E/G can support flat gauge fields with non-trivial holonomy is the basis of the "Hosotani mechanism" for breaking gauge symmetries.

The "quotient Calabi–Yau" manifolds of string theory are special examples of an interesting class of manifolds which we may call the "locally Kählerian" Einstein manifolds. These are Einstein manifolds of the form M^E/G, where M^E is a simply connected, irreducible Kähler–Einstein manifold, and where G acts isometrically but not necessarily holomorphically (so that the *restricted* holonomy group of M^E/G is contained in $U(n)$, where n is the complex dimension of M^E). In this work we present some results on this important class of Einstein manifolds, mainly confining ourselves to the case in which M^E/G is compact and of non-negative Ricci