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Crystal Base and a Generalization of the Littlewood–Richardson Rule for the Classical Lie Algebras

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Abstract. We shall give a generalization of the Littlewood–Richardson rule for $U_q(\mathfrak{g})$ associated with the classical Lie algebras by use of crystal base. This rule describes explicitly the decomposition of tensor products of given representations.

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0. Introduction

In representation theory, it is one of the most fundamental problems to decompose a given representation into the irreducible components. For the Lie algebra $\mathfrak{gl}(n)$, we know a very famous rule called the Littlewood–Richardson rule, which gives the irreducible decomposition of the tensor product of two finite-dimensional irreducible representations. There are various generalizations of this rule to other Lie algebras (e.g. cf. [B-Z, L, T]). The purpose of this paper is to give an explicit description of irreducible decomposition of tensor products of finite-dimensional representations of the q-analogue of universal enveloping algebra associated with the classical Lie algebras by a new tool "crystal base."

The notion of the q-analogue of universal enveloping algebras was introduced by V.G. Drinfeld ([D]) and M. Jimbo ([J]) in 1985 independently. In 1990, the theory of crystal base was constructed by M. Kashiwara ([K1, K2]). Roughly