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Renormalization Group and the Ultraviolet Problem in the Luttinger Model

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Abstract. The Luttinger model describes a non-local interacting relativistic theory for spinless and massless fermions. Albeit the exact solution is already known, the perturbative approach to the model via the renormalization group is useful on account of the connection to the study of more realistic models' behaviour near the Fermi surface. In this work we show that the effective potential describing the interaction on the physical scale p_0^{-1} is analytical in the coupling constants, and has an exponential decay on that scale. Besides the physical motivation of this approach, the problem is also technically interesting, since it is an example of a trivially superrenormalizable theory, as far as the ultraviolet region is concerned; nevertheless the proof is quite delicate, as the convergence of the perturbative series does not follow from the superficial bounds (which would give logarithmic and linear divergences), but is due to accidental compensations furnished by the particular symmetry properties of the model.

Tyger, Tyger, burning bright
In the forests of the night,
What immortal hand or eye
Could frame thy fearful symmetry?

William Blake

Introduction

Recently a considerable effort has been spent in the study of one dimensional models of interacting fermions in the framework of the perturbative theory. The physical reason of this interest is due to the modifications that the interaction brings to the Fermi surface (anomalous Fermi surface) and to the possible relation with the models in higher dimensions. From the mathematical point of view such models, despite their one dimensionality, have a very rich structure because some basic features related to the anomalous behaviour, which can be straightforward to understand in a more qualitative approach, become much more complicated to prove in a rigorous way.