Commun. Math. Phys. 154, 85–97 (1993)

Communications in
Mathematical
Physics

Tetrahedral Zamolodchikov Algebras Corresponding to Baxter's L-Operators

I. G. Korepanov

Pr. Lenina 78-A kv.45, Chelyabinsk, 454080, Russian Federation

Received June 22, 1992

Abstract. Tetrahedral Zamolodchikov algebras are structures that occupy an intermediate place between the solutions of the Yang-Baxter equation and its generalization onto 3-dimensional mathematical physics – the tetrahedron equation. These algebras produce solutions to the tetrahedron equation and, besides, specific "two-layer" solutions to the Yang-Baxter equation. Here the tetrahedral Zamolodchikov algebras are studied that arise from L-operators of the free-fermion case of Baxter's eight-vertex model.

Introduction

The tetrahedron equation is a generalization of the Yang-Baxter equation, which is fundamental in studying the exactly solved models in $1+1$-dimensional mathematical physics, onto the $2 + 1$-dimensional case. Nontrivial solutions of the tetrahedron equation do exist. They were found by Zamolodchikov [1, 2] for the tetrahedron equation "with variables on the faces" and by this author [3, 4] – for the equation "with variables on the links". In the latter case, the solution consists of the commutation relation matrices of the so-called tetrahedral Zamolodchikov algebra – a structure designed to span the gap between the Yang-Baxter and tetrahedron equations.

The existence of a large family of the tetrahedral Zamolodchikov algebras was shown in the papers [3, 4]. However, there are some difficulties here: firstly, the explicit calculation of the commutation relation matrices and, secondly, the verification of whether those matrices really satisfy the tetrahedron equation. These difficulties have been overcome in the mentioned works only in one particular "trigonometrical" case. In addition to the solutions of the tetrahedron equation, the tetrahedral Zamolodchikov algebras produce by themselves the "two-layer" solutions to the Yang-Baxter equation, and here again only the trigonometrical case has been studied [5].

In the present paper, the commutation relation matrices S are calculated in a more general case, with the trigonometrical functions replaced by elliptic ones. The key role