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A Concept of the Mass Center of a System of Material Points in the Constant Curvature Spaces

G. A. Galperin

Department of Mathematics, Northwestern University, Evanston, IL 60208, USA

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Abstract. This article demonstrates that in the Lobatchevsky space and on a sphere of arbitrary dimensions, the concept of the mass center of a system of mass points can be correctly defined. Presented are: a uniform geometric construction for defining the mass center; hyperbolic and spheric "lever rules"; the theorem of uniqueness for determining the mass center in these spaces. Among the compact manifolds, only the sphere possesses this property.

1. Preliminary. Statement of the Main Results

The classical definition of the centroid (A, m) of a system of material points can be stated as follows: a point A with mass m is called *the centroid* of a system of material points $A_1, \dots, A_k$ with masses $m_1, \dots, m_k$ in the Euclidean space $\mathbf{R}^n$ if

$$m \cdot \overrightarrow{OA} = \sum_{i=1}^{k} m_i \cdot \overrightarrow{OA_i}\,, \quad \text{and} \quad m = \sum_{i=1}^{k} m_i\,,$$

where $O \in \mathbf{R}^n$ is an arbitrary point (Fig. 1). Then the mass $m = \sum_{i=1}^{k} m_i$ is located in the point A.

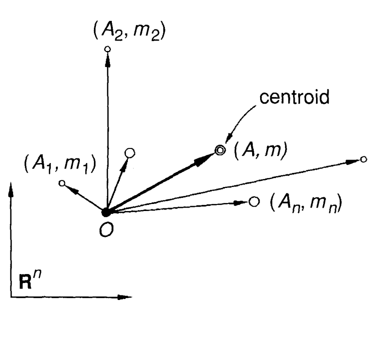

Fig. 1