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Generalized Hypergeometric Functions on the Torus and the Adjoint Representation of $U_q(sl_2)$

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Abstract. We study the homology groups with coefficient in local systems arising in the free field representation of minimal models of conformal field theory on an elliptic curve with punctures. We define an action of the quantum enveloping algebra $U_q(sl_2)$ on a space of relative cycles, extending results obtained previously for the sphere. Absolute cycles are identified with singular vectors. In the case of one puncture, we find that the resulting topological representation is essentially the adjoint representation.

1. Introduction

Recent study [1–3] indicates that there exists a dictionary between homology of certain configuration spaces with coefficients in local systems and representation theory of quantum enveloping algebras [4]. The examples of local systems providing such connections come from integral representation of conformal blocks of conformal field theory [5–14]. The idea is that (in some sense) the charges generating (half of) the quantum group symmetry in the free field representation in conformal field theory are given by integrals over screening operators [15–18]. In a previous paper [2], we have shown the existence of an action of $U_q(sl_2)$ on certain relative locally finite homology groups on configuration spaces on the sphere. In this case, the local system is given by the integrand of the free field representation of conformal blocks of the SU(2) WZW models or minimal models.

In this paper, we consider the situation of the torus, for which one knows explicit integral representations [19–21]. We restrict our attention to the case of minimal models, which is the simplest. The main difference is that the local system is not given by a line bundle as in the case of the sphere, but rather a vector bundle. From the point of view of free fields, this follows from the fact that the space of free field conformal blocks on the torus is higher dimensional.

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