Commun. Math. Phys. 153, 521–557 (1993)

Communications in Mathematical Physics
© Springer-Verlag 1993

Chern-Simons Invariants of 3-Manifolds Decomposed along Tori and the Circle Bundle Over the Representation Space of T^2

Paul Kirk[1] **and Eric Klassen**[2]

[1] Department of Mathematics, Indiana University Bloomington, IN 47405, USA
[2] Department of Mathematics, Florida State University, Tallahassee, FL 32306, USA

Received June 1, 1992

Abstract. We describe a cut-and-paste method for computing Chern-Simons invariant of flat G-connections on 3-manifolds decomposed along tori, especially for $G = SU(2)$ and $SL(2, C)$. We use this method to make computations of $SU(2)$ Chern-Simons invariants of graph manifolds which generalize Fintushel and Stern's computations for Seifert-fibered spaces. We also use this technique to give a simple derivation of a formula of Yoshida relating the flat $SL(2, C)$ Chern-Simons invariant of the holonomy representation to the volume and the metric Chern-Simons invariant for cusped hyperbolic 3-manifolds.

1. Introduction

This paper is a continuation of [KK2]. In that paper we described a method for computing the Chern-Simons invariants of $SU(2)$ representations of a 3-manifold obtained by surgery on a knot K in a closed manifold M in terms of the image of the restriction $R(M - K) \to R(T)$, where $R(X)$ denotes the space of conjugacy classes of representations of the fundamental group of X in $SU(2)$ and T is the boundary torus of $M - K$. The main purpose of this paper is to show how to compute Chern-Simons invariants of a closed manifold in terms of an arbitrary decomposition of the manifold along tori. Cutting a 3-manifold along tori is a useful procedure in 3-manifold theory. In addition to surgery on knots and links, this includes also decompositions along incompressible tori in the sense of Jaco-Shalen and Johannson [J]. This cuts a 3-manifold into simpler pieces, namely Seifert-fibered 3-manifolds and complete hyperbolic 3-manifolds. The basic idea is to define Chern-Simons invariants for a manifold whose boundary consists entirely of tori. We then show how to use these methods to explicitly compute Chern-Simons invariants of various representations of 3-manifolds with toral boundaries, including many Seifert-fibered and hyperbolic manifolds.

Both authors acknowledge support from the NSF