Commun. Math. Phys. 153, 447–463 (1993)

Communications in Mathematical Physics

BRST Model for Equivariant Cohomology and Representatives for the Equivariant Thom Class

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Received March 6, 1992; in revised form August 31, 1992

Abstract. In this paper the BRST formalism for topological field theories is studied in a mathematical setting. The BRST operator is obtained as a member of a one parameter family of operators connecting the Weil model and the Cartan model for equivariant cohomology. Furthermore, the BRST operator is identified as the sum of an equivariant derivation and its Fourier transform. Using this, the Mathai-Quillen representative for the Thom class of associated vector bundles is obtained as the Fourier transform of a simple BRST closed element.

1. Introduction

Recently, there has been much interest in so-called topological quantum field theories (see, e.g., [W1, W2, BS, OSvB]). Examples of these are field theories defined by a Lagrangian $\mathscr{L}$ which results in a constant (or even zero) action $\mathscr{S} = \int \mathscr{L}$. Although these theories do not contain any dynamics, they lead to very interesting expressions for topological and even differential invariants of finite dimensional manifolds. Another reason why one is interested in topological theories is because they give rise to the most simple quantum systems, namely those with finite dimensional Hilbert space.

Also, the BRST method of quantization (see, e.g., [H, KS]) is under intensive investiation, but in the case of field theories this method is far from being understood. Trying to understand the BRST method for the simplest field theories is therefore a natural thing to do.

In the first part of this paper we will show that the BRST-cohomology of certain topological models equals the equivariant cohomology of the configuration space. Although this fact is known and used for some time now, it was never shown in what context it is true. Here, we will show that equality can be proved in a finite

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