Commun. Math. Phys. 153, 431–445 (1993)

Communications in Mathematical Physics

Recovery of Singularities for Formally Determined Inverse Problems

Ziqi Sun[1]* and Gunther Uhlmann[2]**

[1] Department of Mathematics and Statistics, Wichita State University, Wichita, Kansas 67208, USA

[2] Department of Mathematics, University of Washington, Seattle, Washington 98195, USA

Received June 29, 1992

Abstract. In this paper we considered several formally determined problems in two dimensions. There are no global identifiability results for these problems. However, we can recover an important feature of these functions, namely their singularities. More precisely, we prove that one can determine the location and strength of singularities of an L^∞ compactly supported potential by knowing the associated scattering amplitude at a fixed energy. Also we prove that one can determine the location and strength of the singularities of the sound speed of a medium by making measurements just on the boundary of the medium.

1. Introduction and Statement of the Results

In this paper we consider formally determined inverse problems in two dimensions. These problems involve determination of the sound speed of a medium by making measurements at the boundary of the medium or a quantum mechanical potential by making scattering measurements away from the support of the potential.

For the problems under consideration there are no global identifiability results available in the case of a general L^∞ potential or sound speed. The results known are either local ([S-U, Su I, II]) or generic ([Su-U I]). Kohn and Vogelius ([K-V]) proved a global identifiability result in the case that the potential is piecewise analytic. In this paper we consider the problem of determining the strength and location of the singularities of the sound speed or the potential from either boundary measurements or scattering information.

All the results we prove are reduced to prove a similar result for the inverse problem of determining a bounded, measurable potential from the Dirichlet to Neumann map associated to the Schrödinger equation at zero energy. We proceed next to define this map.

* Partially supported by NSF grant DMS-9123742

** Partially supported by NSF grant DMS-9100178