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On the Spectral Problem for Anyons

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Abstract. We consider the spectral problem resulting from the Schrödinger equation for a quantum system of $n \geqq 2$ indistinguishable, spinless, hard-core particles on a domain in two dimensional Euclidian space. For particles obeying fractional statistics, and interacting via a repulsive hard core potential, we provide a rigorous framework for analysing the spectral problem with its multi-valued wave functions.

1. Introduction

Let $\mathscr{M}$ be a bounded domain in $\mathbb{R}^2$, with boundary $\partial\mathscr{M}$ which we assume to be smooth. The standard choice for the configuration space for a system of n indistinguishable particles constrained to the surface $\mathscr{M}$, and satisfying fractional statistics is the manifold

$$Q_n = (\mathscr{M}^n - \delta_n)/S_n . \tag{1.1}$$

Here $\mathscr{M}^n$ denotes the n-fold cartesian product of $\mathscr{M}$ with itself, δ_n denotes the subset of points where two or more particle coordinates coincide (the diagonal) and S_n denotes the group of permutations on n symbols. The fundamental group of Q_n, $\pi_1(Q_n)$ is the n-braid group $B_n(\mathscr{M})$ of $\mathscr{M}$.

Now let $\chi\colon \pi_1(Q_n) \to U(1)$ be a finite, one dimensional, irreducible representation; clearly such a representation is a homomorphism onto the cyclic group of the roots of unity, $U_m = \{\exp(2\pi ik/m),\ k = 0, 1, \ldots (m-1)\}$, for some $m \geqq 1$. Let $\tilde{Q}_n^{[m]}$ be the m-fold covering space of Q_n associated with the representation U_m, with $B_n(\mathscr{M})$ acting as deck transformations, and let $\pi\colon \tilde{Q}_n^{[m]} \to Q_n$, be the natural projection. It has been proposed, [10], that the space of admissible wave functions be a complex Hilbert space obtained from the class of smooth equivariant functions

$$C^\infty_{[m]}(\tilde{Q}_n^{[m]}) = \{\tilde{\psi}\colon \tilde{Q}_n^{[m]} \to \mathbb{C}\colon \tilde{\psi}(\gamma z, \gamma z^*) = \chi(\gamma)\tilde{\psi}(z, z^*), \text{ for all } \gamma \in B_n(\mathscr{M})\} . \tag{1.2}$$

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