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On Concentration of Positive Bound States of Nonlinear Schrödinger Equations

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Abstract. We study the concentration behavior of positive bound states of the nonlinear Schrödinger equation

$$ih\frac{\partial\psi}{\partial t} = \frac{-h^2}{2m}\Delta\psi + V(x)\psi - \gamma|\psi|^{p-1}\psi\,.$$

Under certain condition on V, we show that positive ground state solutions must concentrate at global minimum points of V as $h \to 0^+$; moreover, a point at which a sequence of positive bound states concentrates must be a critical point of V. In case that V is radial, we prove that the positive radial solutions with least energy among all nontrivial radial solutions must concentrate at the origin as $h \to 0^+$.

Section 1. Introduction and Description of Main Results

Of concern are standing wave solutions of the following nonlinear Schrödinger equations:

$$ih\frac{\partial\psi}{\partial t} = \frac{-h^2}{2m}\Delta\psi + V(x)\psi - \gamma|\psi|^{p-1}\psi \quad \text{with } x\in\mathbb{R}^n\,, \tag{1.1}$$

i.e., solutions of the form

$$\psi(x,t) = \exp(iEt/h)u(x)\,, \tag{1.2}$$

where h, m, γ and p are positive constants, $p > 1$, $E\in\mathbb{R}$, V is real and belongs to $C^1(\mathbb{R}^n)$ and u is real. In [FW], Floer and Weinstein proved for small $h > 0$ (and for $p = 3$, $n = 1$) the existence of standing wave solutions concentrating at each given nondegenerate critical point of the potential V, under the condition that V is bounded. In [O_1, O_3], Oh generalized this result and obtained for small $h > 0$ the existence of multi-lump standing wave solutions with u in (1.2) being positive and

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