On the Haar Measure of the Quantum $SU(N)$ Group

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Abstract. We prove that the Haar state associated to the compact matrix quantum group $SU_\mu(N)$ is faithful for $\mu \in\ ] -1, 1[$, $\mu \neq 0$, and any $N \geq 2$.

In [Wo2] the theory of *Compact Matrix Pseudogroups* was initiated and it was shown that many of the fundamental notions of Harmonic Analysis have natural extensions in this context. Among them, probably the most important achievement is the notion of *Haar measure* and the Peter–Weyl type theorem for compact matrix pseudogroups. The next step in this direction was the Tannaka–Krein Duality theorem proved in [Wo3], which made the connection between the objects of Woronowicz and *Quantum Groups* (Drinfeld and Jimbo, see [Dr]) more clear. In fact, a compact matrix pseudogroup, or, more precisely, its algebra of "continuous functions," can be viewed as a completion of the Hopf *-algebra of "coefficients" of representations of a quantum group, cf. [Ro, So1, So2]. Apart from Woronowicz's approach this characterization allowed further investigations on the structure of the corresponding C^*-algebras (see [So1, VS2]).

In this paper we shall apply such a philosophy in order to obtain that the *"support of the Haar measure on $SU_\mu(N)$ is the entire space $SU_\mu(N)$*. The appropriate sense of this statement should be the *faithfulness of the Haar state on the C^*-algebra $C(SU_\mu(N))$*. Recall that a positive functional ϕ on a C^*-algebra is called *faithful* if for any element $a \neq 0$ one has $\phi(a^*a) \neq 0$. This problem was left open in [Wo2], where the faithfulness is shown only on a dense subalgebra. In particular, from our result it follows that the enveloping C^*-norm on $C(SU_\mu(N))$ can simply be given as the GNS norm associated to the Haar state.