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Meromorphic $c = 24$ Conformal Field Theories

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Abstract. Modular invariant conformal field theories with just one primary field and central charge $c = 24$ are considered. It has been shown previously that if the chiral algebra of such a theory contains spin-1 currents, it is either the Leech lattice CFT, or it contains a Kac-Moody sub-algebra with total central charge 24. In this paper all meromorphic modular invariant combinations of the allowed Kac-Moody combinations are obtained. The result suggests the existence of 71 meromorphic $c = 24$ theories, including the 41 that were already known.

1. Introduction

A conformal field theory is characterized by two algebraic structures: the chiral algebra and the fusion algebra. The chiral algebra consists at least of the Virasoro algebra, which in general is extended by other operators of integer conformal weight. The representations or primary fields of the chiral algebra obey a set of fusion rules, determining which primary fields can appear in the operator product of two such fields. In general, both the chiral algebra and the set of fusion rules are non-trivial.

If one is interested in classifying conformal field theories, it seems natural to start with the simplest ones. For example, one might consider theories in which either the chiral algebra or the fusion algebra is as simple as possible. Theories of the former kind form the "minimal series" [1] (whose chiral algebra consists a priori only of the Virasoro algebra) and have been classified completely [2–4]. The theories with the simplest possible set of fusion rules are those with only one primary field **1**, and a fusion rule $\mathbf{1} \times \mathbf{1} = \mathbf{1}$. In such theories the entire non-trivial structure resides obviously in the chiral algebra.

Theories of this kind have extremely simple modular transformation properties [5]. The identity is self-conjugate, and hence the charge conjugation matrix C must be equal to 1. Therefore $S = \pm 1$ and the identity character $\chi(\tau)$ satisfies $\chi\left(-\frac{1}{\tau}\right) = \pm\chi(\tau)$. Choosing $\tau = i$, and noting that the character is a polynomial with positive coefficients in $q = e^{2\pi i \tau}$ so that $\chi(i) \neq 0$, we see that $S = 1$. Furthermore,