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On a Ferromagnetic Spin Chain

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Abstract. The quotient $\zeta(s-1)/\zeta(s)$ of Riemann zeta functions is shown to be the partition function of a ferromagnetic spin chain for inverse temperature s.

Contents

1. Introduction

The aim of this article is to relate ideas and concepts from statistical mechanics to the Riemann zeta function.

The quotient $Z(s) = \zeta(s-1)/\zeta(s)$ is interpreted as the partition function of an infinite ferromagnetic spin chain.

The existence of a connection between number theory and statistical mechanics has been conjectured by Kac (see his Comments in Pólya [4], pp. 424–426), Newman [3], Ruelle [5] and others.

One motivation for that conjecture has been the Lee–Yang circle theorem of statistical mechanics. In its basic form it states that all zeroes of the partition function of a ferromagnetic Ising model in the complex activity plane have unit modulus.