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Super Toeplitz Operators and Non-Perturbative Deformation Quantization of Supermanifolds

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Abstract. The purpose of this paper is to construct non-perturbative deformation quantizations of the algebras of smooth functions on Poisson supermanifolds. For the examples $U^{1|1}$ and $C^{m|n}$, algebras of super Toeplitz operators are defined with respect to certain Hilbert spaces of superholomorphic functions. Generators and relations for these algebras are given. The algebras can be thought of as algebras of "quantized functions," and deformation conditions are proven which demonstrate the recovery of the super Poisson structures in a semi-classical limit.

I. Introduction

I.A. Deformation quantization is a natural scheme for constructing non-commutative spaces, in the sense of [10], as deformations of Poisson manifolds. In this framework, the algebra of functions on a manifold is replaced by a family of non-commuting algebras of "quantized functions," which are indexed by a parameter ("Planck's constant"). The guiding principle of the deformation quantization construction is that the classical algebra of functions is obtained from the quantized algebras in the limit as Planck's constant goes to zero, with a first order correction determined by the Poisson structure on the manifold. This scheme was originally proposed in the context of a formal power series in the deformation parameter [2, 4], but has recently been extended to the non-perturbative setup (see [18, 19], and references therein).

This non-perturbative scheme was applied to the Poincaré disc in [12], with an approach using Toeplitz operators as quantization maps based on the ideas of [3–6] (Toeplitz operators were also used to quantize the sphere in [20]). The techniques of [12] have been applied to compact Riemann surfaces in [13], to the n-dimensional complex vector space in [9], to a two-parameter deformation of the unit disc [14], and to the four large classes of Cartan domains in [8]. The basic

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