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General Integrable Problems of Classical Mechanics

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Abstract. Several classical problems of mechanics are shown to be integrable for the special systems of coupled rigid bodies, introduced in this work and called C^k-central configurations. It is proven that dynamics of an arbitrary C^k-central configuration in a Newtonian gravitational field with an arbitrary quadratic potential is integrable in the Liouville sense and in the theta-functions of Riemann surfaces. Hidden symmetry of the inertial dynamics of these configurations is disclosed and reductions of the Lagrange equations to the Euler equations on Lie coalgebras are obtained. Reductions and integrable cases of a heavy C^k-central configuration rotation around a fixed point are indicated. Separation of rotations of a space station type orbiting system, being a C^k-central configuration of rigid bodies, is proven. This result leads to the possibility of the independent stabilization of rotations of the rigid bodies in such orbiting configurations.

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