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Asymptotic Properties of Generalized Eigenfunctions for Three Body Schrödinger Operators

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Abstract. We study the spatial asymptotics of generalized eigenfunctions of three body Schrödinger operators and derive all the S-matrices with initial state of 2 clusters.

1. Introduction

This paper is a continuation of our previous work [2] and deals with properties of S-matrices for three body Schrödinger operators. We first recall the basic notation and results of [2]. In $\mathbf{R}^3$ we consider three particles with mass m_i and position x^i. Let α be a pair (i, j) and

$$x^\alpha = \sqrt{2m_\alpha}(x^i - x^j), \quad x_\alpha = \sqrt{2n_\alpha}\left(x^k - \frac{m_i x^i + m_j x^j}{m_i + m_j}\right),$$

$$\frac{1}{m_\alpha} = \frac{1}{m_i} + \frac{1}{m_j}, \quad \frac{1}{n_\alpha} = \frac{1}{m_k} + \frac{1}{m_i + m_j}.$$

Then the Schrödinger operator is defined by

$$H = H_0 + \sum_\alpha V_\alpha(x^\alpha), \quad H_0 = -\Delta_{x^\alpha} - \Delta_{x_\alpha} \tag{1.1}$$

on $L^2(X)$, where $X = \{(x^1, x^2, x^3); \sum_{i=1}^3 m_i x^i = 0\}$. We consider wave operators, known to exist when $V_\alpha(x^\alpha)$'s decay faster than $|x^\alpha|^{-1}$,

$$W_0^\pm = \mathrm{s} - \lim_{t \to \pm\infty} e^{itH} e^{-itH_0}, \tag{1.2}$$

$$W_\alpha^\pm = \mathrm{s} - \lim_{t \to \pm\infty} e^{itH} e^{-itH_\alpha} J_\alpha, \tag{1.3}$$

$$H_\alpha = H_0 + V_\alpha, \quad (J_\alpha f)(x^\alpha, x_\alpha) = u_\alpha(x^\alpha) f(x_\alpha), \tag{1.4}$$