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Perturbation Theory of Wightman Functions

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Abstract. A perturbative expansion of the Wightman functions, and more generally of vacuum expectation values of products of time-ordered and anti-time-ordered products, is derived for Φ_4^4 field theory. The result is expressed as a sum over generalized Feynman graphs. The derivation is based exclusively on the equation of motion and the Wightman axioms. Neither canonical commutation relations nor asymptotic conditions are needed at any point. In the zero-mass case the individual graphs are infrared divergent, but the sum over all graphs of a given order is convergent.

1. Introduction

Both the time-ordered functions (τ-functions) and the Wightman functions (W-functions) of a relativistic quantum field theory contain in principle the full information on the theory. This is fortunate because functions, even generalized ones, are in many respects easier to work with than unbounded operators.

In many approximation schemes, especially in perturbation theory (PT), the τ-functions are the objects which are simplest to calculate. They are also of high practical value because of their close connection with the S-matrix. The W-functions, on the other hand, are more useful for examining the basic physical structure of a theory, because their properties are very directly related to the fundamental assumptions like relativistic invariance, spectral properties, and locality. In particular, the reconstruction of the operator formulation of the theory from the W-functions is very transparent. This may be helpful for such problems as the exact characterization of the physical state space of a gauge theory, which problem is closely related to the confinement question. Also, the W-functions are not subject to the notorious ambiguities of the definition of the time-ordered products. It is therefore desirable to have systematic, if only approximate, methods of calculating W-functions.

PT is still one of the most powerful and best understood approximation schemes in quantum field theory. It can give significant insights into the structure of a theory