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# Irreducible Unitary Representations of Quantum Lorentz Group

**Wiesław Pusz**

Department of Mathematical Methods in Physics, Faculty of Physics, University of Warsaw, Hoza 74, PL 00-682 Warsaw, Poland

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**Abstract.** A complete classification of irreducible unitary representations of a one parameter deformation $S_qL(2, C)$ $(0<q<1)$ of $SL(2, C)$ is given. It shows that in spite of a popular belief the representation theory for $S_qL(2, C)$ is not "a smooth deformation" of the one for $SL(2, C)$.

## 0. Introduction

A theory of quantum deformations of the classical locally compact groups still seems far from being complete. According to [16] one can distinguish a purely algebraic (Hopf-algebra or Hopf *-algebra) level, topological ($C^*$-algebra) level and intermediate Hilbert space (i.e. the representation theory) level. For the compact groups the algebraic and topological approaches are equivalent since the topological level is well understood and there is a natural way of passing to it from the algebraic one (see e.g. [14, 15]) via the Hilbert space level. In effect one obtains a smooth deformation of group structure and its representation theory.

This experience is a source of the popular belief that it is also the case for general locally compact groups. A class of Pontryagin duals for compact quantum groups is also well established on the $C^*$-algebra level [8] and seems to confirm this conviction, but for the non-compact case there is no general theory of topological quantum deformation at the moment.

The study of other examples indicates that new phenomena can occur which are not seen on the algebraic level:
– The deformation may not exist on the $C^*$-algebra level (cf. [16] where non-existence of comultiplication for quantum $SU(1,1)$ group for real values of deformation parameter was proved).
– The deformation on the $C^*$-algebra level exists under some additional conditions (e.g. restrictions on spectra of operators involved in the theory (see the spectral condition in [16] for the case of $E(2)$ – the group of motions of the Euclidean plane)).