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Geometry and Integrability of Topological-Antitopological Fusion

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Abstract. Integrability of equations of topological-antitopological fusion (being proposed by Cecotti and Vafa) describing the ground state metric on a given 2D topological field theory (TFT) model, is proved. For massive TFT models these equations are reduced to a universal form (being independent on the given TFT model) by gauge transformations. For massive perturbations of topological conformal field theory models the separatrix solutions of the equations bounded at infinity are found by the isomonodromy deformations method. Also it is shown that the ground state metric together with some part of the underlined TFT structure can be parametrized by pluriharmonic maps of the coupling space to the symmetric space of real positive definite quadratic forms.

Introduction

The idea of topological field theories (TFT) as solvable models without local, propagating degrees of freedom was proposed in [1]. In [1–4] it was shown that topological correlators (at tree level) in a 2D TFT model are holomorphic functions on moduli of the TFT model obeying an overdetermined system of nonlinear PDE (the equations of associativity of primary operator algebra). Integrability of these equations was proved in [5].

The problem of calculation of the ground state metric of a family of TFT was studied in a general situation (for both massless and massive theories) in [6]. In this paper a system of PDE for the ground state metric (being a Hermitian metric on the moduli space of TFT) was derived. The topological and "antitopological" (i.e. complex conjugate) correlators serve as coefficients of these PDE. This general construction of calculating of ground state metric was called in [6] a *topological-antitopological fusion.* The equation of the same form arises for the metric on moduli space of Calabi-Yau varieties [7, 8]. The Hermitian metric on the moduli

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