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Dynamical Entropy of Space Translations of CAR and CCR Algebras with respect to Quasi-free States

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Abstract. We compute the dynamical entropy in the sense of Connes, Narnhofer and Thirring of space translations of the CAR and CCR algebras in ν-dimensional continuous spaces with respect to invariant quasi-free states. It turns out that the dynamical entropies are equal to the corresponding mean entropies of the systems under consideration. Computing the mean entropies explicitly we derive the entropy formulas for the systems.

1. Introduction

In their recent paper [7] Connes, Narnhofer and Thirring extended the notion of a dynamical entropy of classical dynamical systems introduced by Kolmogorov and Sinai [9, 18] to the case of automorphisms of C^*-algebras invariant with respect to a given state. The dynamical entropy is the maximal entropy increase of a subalgebra per unit time and measure how chaotically the system evolves. As in the classical ergodic theory [5], the concept of the entropy should be mathematically useful to find a classification of quantum chaotic evolutions. Some attempts have been undertaken in this direction [10, 12, 13].

In order to find a classification of automorphisms of C^*-algebras, it should be important to develop the methods which enable to compute the entropy for quantum systems. There have been some results in this field. The dynamical entropies of space translation for the Gibbs state of one dimensional bounded quantum lattice systems [7, 11] and the quasi-free evolutions of the CAR algebras [19] have been computed by utilizing continuity properties of the entropy [7, 19]. Recently the chaotic behavior of automorphisms on the rotation algebra [10] and noncommutative 2-shift [13] were investigated. In [16] we have extended the continuity [7] of the Kolmogorov-Sinai type for AF-algebras to non-AF situations and applied it to the unbounded quantum spin

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