

Renormalization for Golden Circles[★]

Andreas Stirnemann

Department of Mathematics, University of Exeter, Laver Building, North Park Road,
Exeter EX4 4QE, UK

Received February 18, 1992; in revised form August 10, 1992

Abstract. We prove that twist maps of the cylinder that are attracted by any fixed point of MacKay’s renormalization operator have a transitive invariant golden circle, provided the fixed point satisfies a few simple, purely topological conditions. These conditions can be verified by finite-precision arithmetics; they are fulfilled for the simple fixed point and seem to be fulfilled for the critical fixed point. Taking existence and hyperbolicity of the critical fixed point for granted, we conclude that the standard map has a critical invariant golden circle; the induced map on the circle is topologically conjugate to a rigid rotation; we can show that the conjugator is Hölder continuous; moreover, it is not differentiable on a dense set of points.

Contents

0. Introduction	369
1. Domain Pairs	380
2. Fibonacci Strings	384
3. Approximate Invariant Sets	387
4. Asymptotic Self-Similarity	389
5. The Necklace	396
6. The Golden Curve	400
7. Ordering the Patches	407
8. Appendix	422

0. Introduction

0.1. Statement of Results. Let us consider one-parameter-families of symmetric area-preserving twist maps of the cylinder. A well known example is the *standard family*, given by

$$x' = x + y', \quad y' = y - \frac{\kappa}{2\pi} \sin 2\pi x.$$

[★] This paper is part of a PhD thesis that is in preparation under the supervision of Oscar E. Lanford III at the ETH. I thank Oscar Lanford for having asked me precisely the right questions