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A Family of Poisson Structures on Hermitian Symmetric Spaces

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Abstract. We investigate the compatibility of symplectic Kirillov-Kostant-Souriau structure and Poisson-Lie structure on coadjoint orbits of semisimple Lie group. We prove that they are compatible for an orbit compact Lie group iff the orbit is hermitian symmetric space. We prove also the compatibility statement for non-compact hermitian symmetric space. As an example we describe a structure of symplectic leaves on $\mathbf{CP}^n$ for this family. These leaves may be considered as a perturbation of Schubert cells. Possible applications to infinite-dimensional examples are discussed.

1. Introduction

It became clear recently that quantum groups play a fundamental role in modern field-theoretical models. An interpretation of quantum groups as new symmetries of chiral conformal field theory was discussed in [AGS] and [FG]. A quasiclassical counterpart of quantum groups, Poisson-Lie groups was identified with classical symmetries of classical chiral theory [F, F-G].

By a well-known prescription of Kirillov-Kostant-Souriau coadjoint orbits $\mathcal{O}$ of a compact semisimple Lie group G have a natural symplectic structure and hence are Poisson manifolds. The developments of the Poisson-Lie group theory [ST, W] make it possible to introduce another Poisson structure on $\mathcal{O}$ which is induced from the Poisson-Lie structure on the compact semisimple Lie group G. We denote these structures by π_{Kir} and $\pi_{\text{P-L}}$, respectively. The main result of this paper is to show that π_{Kir} and $\pi_{\text{P-L}}$ are compatible on the orbit $\mathcal{O}$ iff $\mathcal{O}$ is a hermitian compact symmetric space (h.c.s.s.) occurring in the Cartan list.

On the other side, using the duality between compact and non-compact symmetric spaces, we can show that $\pi_{\text{P-L}}$ and an imaginary part of the canonical hermitian form are compatible for non-compact hermitian symmetric spaces. The consideration of symplectic leaves of the family $\pi_{\text{Kir}}+\pi_{\text{P-L}}$ in $\mathbf{C}P^n$ provides an interesting example of the deformation of the Schubert cells (see Sect. 7).