Commun. Math. Phys. 152, 269–297 (1993)

Communications in Mathematical Physics

A Polyakov Action on Riemann Surfaces (II)

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Received December 6, 1991; in revised form September 20, 1992

Abstract. We continue the model independent study of the Polyakov action on an arbitrary compact surface without boundary of genus larger than 2 as the general solution of the relevant conformal Ward identity. A general formula for the Polyakov action and an explicit calculation of the energy-momentum tensor density is provided. It is further shown how Polyakov's SL(2,C) symmetry emerges in a curved base surface.

1. Introduction

Since the mid-eighties, a large body of theoretical and mathematical literature has been devoted to the study of two-dimensional conformal field theories on Riemann surfaces without boundary [1, 2]. These models are relevant in string theory and in the analysis of the 2-dimensional statistical system obeying certain periodic boundary conditions at criticality. In more recent times, the dependence on the background geometry has been exploited to obtain effective actions for two-dimensional quantum gravity. This has led to exciting developments in non-critical string theory [3, 4] and may conceivably shed some light on the quantization programme of higher dimensional gravity.

Most of the studies on the subject are concerned with Lagrangian field theories on a two-dimensional Riemannian manifold (Σ, g) which are both Weyl and diffeomorphism invariant at the classical level [5–7]. The quantization program is carried out by means of a diffeomorphism invariant renormalization scheme, typically the ζ function scheme. In general, however, a Weyl anomaly is produced in this way. Let us see this in greater detail. Parametrize the metric g as usual as

$$g = \exp(\phi)\varrho_0|dz + \mu d\bar{z}|^2 \tag{1.1}$$

[8–10]. Here, $z, \bar{z}$ are the coordinates of a reference holomorphic coordinate covering. ϕ is the Weyl phase. μ is the Beltrami differential characterizing the conformal class

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