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Are There Chaotic Tilings?

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Abstract. We develop a class of examples in the form of tiling dynamical systems for use as toy models in statistical mechanics, to analyze the possible existence of disordered crystals. We give the first such models which are disordered in the sense of having no discrete spectrum.

1. Introduction

Ten years ago, Ruelle published the paper "Do turbulent crystals exist?" [7], in which he suggested the existence of real materials which in thermal equilibrium at low temperature would be quite different microscopically from the usual periodic crystals; the suggested difference would be demonstrated by a diffraction spectrum which was absolutely continuous, even at zero temperature.

Ruelle's argument was based on a comparison of the usual classical statistical mechanical formalism with a typical dynamical system with $\mathbf{R}^3$ action ($\mathbf{R}^3$ representing spatial translations), but without any detailed consideration of the structural role played by interacting particles in the former.

The present paper is motivated by the same problem, but with a different premise. We have chosen to concentrate on the special features which may be due to the role played by the interacting particles in statistical mechanics, with the aim to determine the qualitative low temperature features of generic classical statistical mechanical models with short range interactions. It is well known [7] that no such model has ever been proven to exhibit an ordered (crystalline) phase; presumably the reason is the difficulty in analyzing such models. To obtain results we first restrict attention to zero temperature, and then we distort the models to that of tiling dynamical systems (defined below), as is sometimes done in analyzing quasicrystals [8]. (Roughly speaking, in a tiling dynamical system the phase space

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