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A Global Attracting Set for the Kuramoto-Sivashinsky Equation

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Abstract. New bounds are given for the L^2-norm of the solution of the Kuramoto-Sivashinsky equation

$$\partial_t U(x,t) = -(\partial_x^2 + \partial_x^4)U(x,t) - U(x,t)\partial_x U(x,t) ,$$

for initial data which are periodic with period L. There is no requirement on the antisymmetry of the initial data. The result is

$$\limsup_{t\to\infty} \|U(\cdot,t)\|_2 \leq \text{const. } L^{8/5} .$$

1. Introduction

In this paper, we prove new bounds on the Kuramoto-Sivashinsky equation (KS) by extending the ingenious method of Nicolaenko, Scheurer, and Temam [NST]. We study the KS-equation in its "derivative form:"

$$\partial_t U(x,t) = -(\partial_x^2 + \partial_x^4)U(x,t) - U(x,t)\partial_x U(x,t) . \tag{1.1}$$

The "original equation" is for the integral, $H(x,t) = \int_0^x d\xi U(\xi,t)$. Before we start with the bounds, we give some background material. The interest in the KS-equation is based on its relation as a phase equation for hydrodynamic problems, see Manneville [M] for a derivation. We consider the equation on the interval $[-L/2, L/2]$, with periodic boundary conditions. Since U should be thought of as the derivative of a periodic function, we always require $\int_{-L/2}^{L/2} U = 0$. In the paper [NST] it is shown that if the initial data are in L^2, and are *antisymmetric with respect to the origin,* then the evolution leaves them in L^2, forever, and there is a global attracting set whose diameter in L^2 is bounded. This bound depends on the size L of the system, and the bound given by [NST] is