Commun. Math. Phys. 152, 191–202 (1993)

Communications in Mathematical Physics

Topological Models on the Lattice and a Remark on String Theory Cloning

C. Bachas[1] **and P. M. S. Petropoulos**[2]

[1] Centre de Physique Théorique, Ecole Polytechnique*, F-91128 Palaiseau Cedex, France
[2] Service de Physique Théorique de Saclay**, F-91191 Gif-sur-Yvette Cedex, France

Received May 14, 1992

Abstract. The addition of a topological model to the matter content of a conventional closed-string theory leads to the appearance of many perturbatively-decoupled space-time worlds. We illustrate this by classifying topological vertex models on a triangulated surface. We comment on how such worlds could have been coupled in the Planck era.

1. Many Worlds in String Theory

Topological quantum field theories [1, 2] are characterized by their invariance under local smooth deformations of the background metric. Thus adding a two-dimensional topological model to the matter content of a conventional critical closed-string theory should not affect the decoupling of the Liouville mode and hence also the theory's consistency. Could this then imply that critical string theory is not unique?

In order to address this question we must specify more precisely what we mean by topological models. One way to define them, following Atiyah [3], is through a set of axioms. The basic data is a finite-dimensional space $\mathscr{H}$ of states created by local field operators $\{\phi_1 \equiv \mathbf{I}, \phi_2, \ldots, \phi_M\}$, together with their (symmetric) two- and three-point functions on the sphere:

$$\langle \phi_a \phi_b \rangle_{\text{sph}} = \eta_{ab} \, , \qquad \langle \phi_a \phi_b \phi_e \rangle_{\text{sph}} = c_{abe} \, . \tag{1}$$

The two-point function $\eta_{ab} \equiv c_{1ab}$ must define a non-singular bilinear inner product, which identifies $\mathscr{H}$ with its dual: $(\phi_a)^* \equiv \phi^a = \eta^{ab}\phi_b$. Here and in the sequel indices are raised with the inverse metric η^{ab} and repeated indices are implicitly summed. Unitarity requires the correlation functions of self-adjoint operators to be real. Using the three-point functions and the metric, we can give $\mathscr{H}$ the structure of a *commutative* operator algebra

$$\phi_a \times \phi_b = c_{ab}{}^{e} \phi_e \, . \tag{2}$$

* Laboratoire Propre du Centre National de la Recherche Scientifique UPR A.0014
** Laboratoire de la Direction des Sciences de la Matière du Commissariat à l'Energie Atomique