

Quantum Dressing Orbits on Compact Groups

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Abstract. The quantum double is shown to imply the dressing transformation on quantum compact groups and the quantum Iwasawa decomposition in the general case. Quantum dressing orbits are described explicitly as $*$ -algebras. The dual coalgebras consisting of differential operators are related to the quantum Weyl elements. Besides, the differential geometry on a quantum leaf allows a remarkably simple construction of irreducible $*$ -representations of the algebras of quantum functions. Representation spaces then consist of analytic functions on classical phase spaces. These representations are also interpreted in the framework of quantization in the spirit of Berezin applied to symplectic leaves on classical compact groups. Convenient “coherent states” are introduced and a correspondence between classical and quantum observables is given.

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0. Introduction

Quantum groups were recently introduced by Drinfel’d [7], Jimbo [10], and Woronowicz [30]. In Woronowicz’s approach a compact quantum group is regarded as a

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