Commun. Math. Phys. 152, 29–40 (1993)

Communications in
Mathematical
Physics

Integrability in the Theory of Schrödinger Operator and Harmonic Analysis

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Received June 15, 1992

Abstract. The algebraic integrability for the Schrödinger equation in $\mathbb{R}^n$ and the role of the quantum Calogero–Sutherland problem and root systems in this context are discussed. For the special values of the parameters in the potential the explicit formula for the eigenfunction of the corresponding Sutherland operator is found. As an application the explicit formula for the zonal spherical functions on the symmetric spaces SU_{2n}^*/Sp_n (type A II in Cartan notations) is presented.

Introduction

The discovery of the method of inverse scattering transformation in the sixties began a new era in the history of integrable systems. The stormy development of the theory of integrability influenced various domains of mathematics and mathematical physics. First of all it is concerned with the spectral theory. In particular, in 1974–76 there were discovered the beautiful results in the spectral theory of the Schrödinger operator

$$L = -\frac{d^2}{dx^2} + u(x) \tag{1}$$

with a periodic potential $u(x)$ (see [1] and references there). It turned out that such a spectral property of L as finite-gapness is equivalent to the existence of commuting differential operator A of odd order:

$$[L, A] = 0, \quad A = \frac{d^{2n+1}}{dx^{2n+1}} + \dots, \tag{2}$$

or the existence of the eigenfunction ψ, which is determined on the algebraic curve (see [1]).

This example demonstrates very well the phenomenon, which we would like to call as the integrability in the theory of the Schrödinger operator. In the present paper we continue the investigations of the multidimensional case begun in [2].