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Global Regularity for Vortex Patches**

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Abstract. We present a proof of Chemin's [4] result which states that the boundary of a vortex patch remains smooth for all time if it is initially smooth.

1. Introduction

A vortex patch is a domain D (simply connected, open and bounded) in the Euclidean plane $\mathbf{R}^2$ which moves with a velocity given at each instant of time by

$$v(x) = \nabla^{\perp}\Psi(x) = \begin{pmatrix} -\dfrac{\partial \Psi}{\partial x_2} \\ \dfrac{\partial \Psi}{\partial x_1} \end{pmatrix} \tag{1.1}$$

with

$$\Psi(x) = \frac{\omega_0}{2\pi} \int_D \ln|x - y| dy , \tag{1.2}$$

where the constant ω_0 is time independent. Vortex patches are particular examples of weak solutions of the two dimensional incompressible Euler equation. Well-known results of Yudovich [10] provide a framework for the vortex patch problem. If an initial domain D_0 and a constant ω_0 are prescribed then there exists a unique vortex patch $D(t)$, defined for all time $t \in \mathbf{R}$ which starts out at $t = 0$ from D_0: $D(0) = D_0$. The area of $D(t)$ is constant in time but, in general, other geometric features behave in a less regular fashion. In particular, the length or curvature of the boundary may grow rapidly [1, 6]. If the boundary is smooth enough (C^1), then the velocity is a contour integral on the boundary, and so the problem can be expressed as a self deforming curve in the plane [11] (the curve is the patch

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