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Gelfand–Dikii Analysis for $N=2$ Supersymmetric KdV Equations

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Abstract. We generalize the resolvent approach of Gelfand and Dikii to the KdV equation to study the $N = 2$ supersymmetric KdV equations of Laberge and Mathieu. For the associated Lax operators, we study the coincidence limits of the resolvent kernel and its derivatives, and obtain differential equations which they satisfy. These allow us to obtain recursion relations for the analogues of the Gelfand–Dikii polynomials and to obtain a proof of Hamiltonian integrability of the supersymmetric KdV equations. We are also able to write the Lax equations for the corresponding hierarchies in terms of these polynomials.

1. Introduction

The theory of the Korteweg de Vries (KdV) equation is by now very well understood. It is one of the simplest completely integrable systems available, and plays a role in many areas of physics and mathematical physics. Recently, the importance of KdV equation in the context of conformal field theory and 2D quantum gravity has been recognized [1]. In the matrix model approach to 2D quantum gravity the Gelfand–Dikii polynomials and the recursion relations they obey play a particularly important role.

Supersymmetric generalizations of the KdV equation have also been of interest recently [2, 3]. One might expect that they will be relevant to the study of superconformal field theory and 2D supergravity. In a recent paper [5], the Gelfand–Dikii polynomials and their recursion relations for the $N = 1$ supersymmetric KdV equation have been worked out. In this paper, we consider the extension of this analysis to $N = 2$ supersymmetric KdV equations.

We will first briefly review the situation of the ordinary KdV equation. Our treatment follows that of Gelfand and Dikii [6] and is centred around the resolvent for the Lax operator. The resolvent is an important object because the KdV

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