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Long Range Scattering for Non-Linear Schrödinger and Hartree Equations in Space Dimension $n \geq 2$

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Abstract. We consider the scattering problem for the non-linear Schrödinger (NLS) equation with a power interaction with critical power $p = 1+2/n$ in space dimensions $n = 2$ and 3 and for the Hartree equation with potential $|x|^{-1}$ in space dimension $n \geq 2$. We prove the existence of modified wave operators in the L^2 sense on a dense set of small and sufficiently regular asymptotic states.

1. Introduction

This paper is devoted to the study of the asymptotic behaviour in time of the solutions of the non-linear Schrödinger (NLS) equation and of the Hartree equation

$$i\partial_t u = -(1/2)\Delta u + f(u) \tag{1.1}$$

in the Coulomb like limiting case, in space dimension $n \geq 2$. The non-linear interaction term is

$$f(u) = \lambda |u|^{p-1} u \tag{1.2}$$

with $p - 1 = 2/n$ in the NLS case and

$$f(u) = (V * |u|^2)u = \lambda(|x|^{-1} * |u|^2)u \tag{1.3}$$

in the Hartree case with Coulomb potential $V(x) = \lambda|x|^{-1}$. Here u is a complex function defined in $n + 1$ dimensional space-time, ∂_t denotes the time derivative, Δ denotes the Laplace operator in $\mathbb{R}^n$, and λ is a real constant which matters only through its sign. This paper is the continuation of a previous paper by one of us [17] where the same problem was considered for the NLS equation (1.1), (1.2) in space dimension $n = 1$. We refer to the introduction of [17] for general information on the problem in the 1-dimensional case.

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