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# Boundedness for Large $|x|$ of Suitable Weak Solutions of the Navier-Stokes Equations with Prescribed Velocity at Infinity

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**Abstract.** We consider time-dependent perturbations $u$ of R. Finn's stationary PR-solution of the Navier-Stokes equations, which converges to a constant vector $v_\infty$ as $|x| \to \infty$. For a given time interval $[\delta, T]$, we find a radius $K$ such that $u$ is essentially bounded on $[\delta, T] \times \{|x| \geq K\}$.

## 1. Introduction

We want to investigate the boundedness for large $|x|$ of weak solutions $v$ of the Navier-Stokes system

$$\begin{gathered}
v_t - \Delta v + (v \cdot \nabla) v + \nabla p = f\,, \\
\operatorname{div} v = 0 \quad \text{in} \quad [0, T] \times \Omega\,, \\
v(0, x) = v_0(x) \quad \text{for} \quad x \in \Omega\,, \\
v(t, x) = 0 \quad \text{for} \quad (t, x) \in [0, T] \times \partial\Omega\,, \\
v(t, x) \to v_\infty \quad \text{as} \quad |x| \to \infty\,, \qquad t \in [0, T]\,,
\end{gathered} \tag{1}$$

where $\Omega$ is a smooth exterior domain in $\mathbf{R}^3$, $\operatorname{div} f = 0$, $\operatorname{div} v_0 = 0$, $v_0|\partial\Omega = 0$, $v_0 \to v_\infty$ at infinity, $v_\infty \in \mathbf{R}^3$ is the prescribed constant velocity at infinity.

Most of the previous work concentrates on the case $v_\infty = 0$, where suitable weak solutions of (1) are known to become small in some average sense and bounded for large $|x| \cdot t$, if $f \to 0$ $(t, |x| \to \infty)$, see [CKN, MP, SW]. This means that singularities may occur only in a compact subset of $[0, \infty) \times \Omega$. Some important results are also surveyed in [W].

If $v_\infty \neq 0$ it is not apparent, whether a global weak solution to (1) will converge to a stationary solution as $t \to \infty$. This seems to happen in general only under some smallness assumptions on a corresponding stationary solution, see Miyakawa and Sohr [MS] and Masuda [MK].

In this note we will only assume the existence of a "reasonable" stationary solution $\overset{(0)}{v}$, we will not require any additional smallness. For the existence of $\overset{(0)}{v}$ we refer to