Commun. Math. Phys. 151, 543–576 (1993)

Communications in Mathematical Physics

Global Existence and Asymptotic Behavior of Solutions for the Maxwell–Schrödinger Equations in Three Space Dimensions

Yoshio Tsutsumi

Department of Mathematical Sciences, University of Tokyo, Hongo, Tokyo 113, Japan

Received March 19, 1992; in revised form July 15, 1992

Dedicated to Professor R. Iino on his 70th birthday

Abstract. In this paper we study the global existence and asymptotic behavior of solutions for the Maxwell–Schrödinger equations under the Coulomb gauge condition in three space dimensions with the final states given at $t = +\infty$. This leads to the construction of the modified wave operator for certain scattered data. It is also shown that for the initial data in the range of the modified wave operator, the initial value problem of the Maxwell–Schrödinger equations has the global solutions in time.

1. Introduction and Main Results

In the present paper we consider the global existence and asymptotic behavior of solutions for the Maxwell–Schrödinger equations under the Coulomb gauge condition in three space dimensions:

$$\frac{\partial^2}{\partial t^2}A - \Delta A = -i\{\bar{\psi}(\nabla - iA)\psi - \psi(\nabla + iA)\bar{\psi}\} - \frac{1}{2\pi}\frac{\partial}{\partial t}\nabla\left(\frac{1}{|x|} * |\psi|^2\right), \quad t > 0,\ x \in \mathbf{R}^3, \tag{1.1}$$

$$2i\frac{\partial}{\partial t}\psi + (\nabla - iA)^2\psi - \frac{1}{4\pi}\left(\frac{1}{|x|} * |\psi|^2\right)\psi = 0, \quad t > 0,\ x \in \mathbf{R}^3, \tag{1.2}$$

$$\operatorname{div} A = 0, \quad t \geqq 0,\ x \in \mathbf{R}^3, \tag{1.3}$$

where $*$ denotes the convolution with respect to the spatial variables. Here, $A(t, x)$ is a function from $[0, \infty) \times \mathbf{R}^3$ to $\mathbf{R}^3$ which denotes the electromagnetic real vector potential, and $\psi(t, x)$ is a function from $[0, \infty) \times \mathbf{R}^3$ to $\mathbf{C}$ which denotes the complex scalar field of nonrelativistic charged particles. Equations (1.1)–(1.3) are the classical approximation to the quantum field equations for an electrodynamical nonrelativistic many body system.