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Equivalence of Extrinsic and Intrinsic Quantization for Observables not Preserving the Vertical Polarization

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Abstract. The equivalence of Dirac quantization and intrinsic quantization for arbitrary observables not preserving the vertical polarization is examined for systems with first class constraints that may be considered as the vanishing of the momentum map to a lifted group action. Using a generalized Weyl ordering prescription applicable to arbitrary cotangent bundles we derive necessary and sufficient conditions for the equivalence of the two approaches for different classes of functions. A strong obstruction is found if one requires equivalence for all invariant functions, essentially only admitting trivial bundles. By a restriction to an adequate class of "strongly admissible functions", equivalence can always be obtained in the case of a free group action. Implications for the case of non-free actions and the dependence on the particular quantization scheme are discussed.

1. Introduction

Systems with first class constraints can be quantized in essentially two conceptually different ways: One may impose the constraints classically, divide out the gauge transformations generated by the constraints and quantize the resulting unconstrained system ("intrinsic quantization"). On the other hand, one may try to quantize the original system without the constraints and then impose the constraints as conditions on the physical states ("extrinsic quantization," in particular Dirac quantization [4]).

Normally, first class constraints arise as a consequence of "redundancy symmetries" of the Lagrangian of the theory, i.e., the presence of time-dependent symmetry transformations which do not relate different physical states but different redundant descriptions of the same state (as in gauge theories, where true physical states correspond to gauge equivalence classes of gauge potentials). If this symmetry is supposed to be a redundancy symmetry on the quantum mechanical level as well, the intrinsic method is conceptually preferable. On the other hand, the extrinsic quantization scheme is often preferable from a practical, computational