Commun. Math. Phys. 151, 487–514 (1993)

Communications in
Mathematical
Physics

Complex Quantum Group, Dual Algebra and Bicovariant Differential Calculus

Ursula Carow-Watamura[1] **and Satoshi Watamura**[2]

[1] Department of Physics, Faculty of Science, Tohoku University, Sendai 980, Japan
[2] Department of Physics, College of General Education, Tohoku University, Sendai 980, Japan

Received January 11, 1992; in revised form July 15, 1992

Abstract. The method used to construct the bicovariant bimodule in ref. [CSWW] is applied to examine the structure of the dual algebra and the bicovariant differential calculus of the complex quantum group. The complex quantum group $\mathrm{Fun}_q(SL(N, C))$ is defined by requiring that it contains $\mathrm{Fun}_q(SU(N))$ as a subalgebra analogously to the quantum Lorentz group. Analyzing the properties of the fundamental bimodule, we show that the dual algebra has the structure of the twisted product $\mathrm{Fun}_q(SU(N)) \tilde{\otimes} \mathrm{Fun}_q(SU(N))^*_{\mathrm{reg}}$. Then the bicovariant differential calculi on the complex quantum group are constructed.

0. Introduction

The question of whether the physics of microscopic scale such as the Planck scale is incorporated by the noncommutativity, which is different from the one provided by the quantization of the field theory, has concerned physicists already for a long time.

Such a theory may be described by a noncommutative algebra belonging to a wider class than the one which physicists are now handling. To consider such a possibility, we need to understand more about the general structure of the noncommutative algebra. From this point of view, the quantum group, a class of noncommutative Hopf algebra found in the investigation of the integrable systems is a very interesting example [Dri, Jim, Wor1]. Imposing the covariance under the quantum group, we can also get some other examples of the noncommutative algebra such as the algebra of the comodule, i.e. the quantum space (quantum plane) introduced by Manin [Manin, RTF]. These algebras give some interesting examples of noncommutative algebras such as the quantum Lorentz group [PW,CSSW] and the quantum Poincaré group [LNRT,LNR]. The construction of the noncommutative differential calculus on the quantum group [Wor2,Rosso,Stach,Jur,Weich,MNW,MH,CSWW] and also on the quantum space [Wess,WZ,Zumino,Pusz,CSW,Schm] shows us various promising features peculiar to the noncommutative algebra.