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When is an Interacting Particle System Ergodic?

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Abstract. We consider stochastic flip dynamics for an infinite number of Ising spins on the lattice $\mathbb{Z}^d$. We find a sequence of constructive criteria for the system to be exponentially ergodic. The main idea is to approximate the continuous time process with discrete time processes (its *Euler polygon*) and to use an improved version of previous results [MS] about constructive ergodicity of discrete time processes.

1. Introduction

Ever since the appearance in probability theory of random processes defined through the interaction of infinite interacting particle systems ([S, V, D]), such general questions as in the title have been investigated. The problem is to consider a broad family of systems or automata containing a large number of interacting components and to give criteria under which their behavior is essentially the same as for non-interacting processes. As observed already by Tolstoi (and quoted in [DS]), there are general reasons for "happiness" in a family. Here, this is called **ergodic** behavior and, depending on the context, also stands for *unreliability, memory loss, convergence to the unique invariant measure, high noise regime, absence of phase transition* and more of that. Its counterpart in equilibrium statistical mechanics is the so-called *high temperature regime*, for which the technique of cluster expansions and Dobrushin–Shlosman analysis have given complete characterization.

As is often the case, giving general answers and complete descriptions is often limited to cases of less interest. Critical behavior, catastrophes and bifurcations, coexistence of phases and all that, are in some sense perpendicular to the ergodic

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