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Isospectral Deformations of Random Jacobi Operators

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Summary. We show the integrability of infinite dimensional Hamiltonian systems obtained by making isospectral deformations of random Jacobi operators over an abstract dynamical system. The time 1 map of these so called random Toda flows can be expressed by a QR decomposition.

1. Introduction

Toda systems have been studied extensively since their discovery by Toda in 1967. Since then, several approaches for their integration have been found and many generalizations have been invented.

Examples are:

• The *tied* or *aperiodic Toda* lattice describes isospectral deformations of finite dimensional aperiodic tridiagonal Jacobi matrices. The integration is performed by taking a spectral measure as the new coordinate. From this measure, the matrix can be recovered. The measure moves linearly by the Toda flow. For a generic Hamiltonian, the matrices converge to diagonal matrices for $t \to \pm\infty$. The integration of the first flow, which has an interpretation of particles on the line, has first been performed in [Mo 1]. For the other flows see [DNT]. There are Lie algebraic generalizations of this Toda lattice [Bog 3,K, Sy 2] and interpretations as a geodesic flow [P] or a constrained harmonic motion [DLT].

• The *half infinite Toda lattice* is an infinite dimensional generalization of the tied lattice. It describes isospectral deformations of tridiagonal operators on $l^2(\mathbb{N})$. The integration is technically more difficult and has been performed in [DLT 1], [Ber]. It resembles the integration of the tied lattice. Again, the operators converge in general to diagonal operators [DLT 1].

• The *periodic Toda lattice* consists of isospectral deformations of periodic Jacobi matrices. The first flow describes a periodic chain of particles interacting with an exponential potential. The explicit integration uses methods of algebraic geometry [vM 1,vM 2]. The flow is conjugated to a motion of an auxiliary spectrum. Jacobi's