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Partially $U(1)$ Compactified Scalar Massless Field on the Compact Riemann Surface and the Bosonic String Amplitudes

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Abstract. The theory of the partially $U(1)$ compactified scalar massless field on the compact Riemann surface with Nambu-Goto action is defined. The partition function is determined completely by a choice of the finite-dimensional approximations. The correlation functions are the only correctly defined objects of the theory. The averages of the correlation function asymptotic values provide the amplitudes. For the compact Riemann surfaces of any genus the usual bosonic string amplitudes are the special cases of the above amplitudes.

1. Introduction

Let M be a compact orientable surface of genus g endowed with the Riemannian metric $g_{ij}(x)$, $i, j=1,2$. In the bosonic string theory the Nambu-Goto action for the scalar massless fields $X^\mu(x)$, $\mu=1,\dots,D$ on the surface M is given by

$$S(X^\mu)=-1/2\alpha^2 \int_M d_2x(\det g_{ij}(x))^{1/2} \sum_{i,j=1}^{2} \sum_{\mu=1}^{D} g^{ij}(x)\frac{\partial X^\mu}{\partial x^i}\frac{\partial X^\mu}{\partial x^j}, \tag{1}$$

where $g^{ij}(x)$ is the inverse matrix for the metric matrix $g_{ij}(x)$. It was shown [1, 2] that in the partition function

$$Z=\sum_{g=0}^{\infty} \int Dg_{ij}(x)DX^\mu(x)\exp[S(X^\mu)] \tag{2}$$

for the space dimension $D=26$ the integration over the metrics $g_{ij}(x)$ is reduced to the integration over the complex structure parameters of the Riemann surfaces M. The bosonic string amplitudes are the special correlation functions defined in the following way [3, Vol. 1, Sect. 1.4.2]:

$$\begin{gathered} 1/Z\sum_{g=0}^{\infty} \int Dg_{ij}(x)DX^\mu(x)\left(\prod_{l=1}^{N} V_l(k_l)\right)\exp[S(X^\mu)], \\ V(k)=\int_M d^2x(\det g_{ij}(x))^{1/2}v(x)\exp[i(k,X(x))], \end{gathered} \tag{3}$$