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Existence of Infinitely-Many Smooth, Static, Global Solutions of the Einstein/Yang-Mills Equations

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Abstract. We prove the existence of infinitely-many globally defined singularity-free solutions, to the EYM equations with $SU(2)$ gauge group. The solutions are indexed by a coupling constant, have distinct winding numbers, and their corresponding Einstein metrics decay at infinity to the flat Minkowski metric. Each solution has a finite (ADM) mass; these masses are derived from the solutions, and are *not* arbitrary constants.

1. Introduction

The principal result in this paper is a proof of the existence of a countable set of singularity-free solutions to the coupled Einstein/Yang-Mills (EYM) equations with $SU(2)$ gauge group. These solutions are indexed by a coupling constant, have distinct winding numbers, and their corresponding Einstein metrics decay at infinity to the flat Minkowski metric. Furthermore, we prove that each solution has finite (ADM) mass (cf. [3]). These "masses" are derived from the solutions; they are *not* arbitrary constants.

Our existence proof confirms numerical observations made by Bartnik and McKinnon in [1]. It also extends the result in [2], where the existence of one such solution was established.

The coupled EYM equations with gauge group G can be written in the form

$$R_{ij}-\tfrac{1}{2}Rg_{ij}=\sigma T_{ij}, \qquad d^*F_{ij}=0.$$

Here T_{ij} is the stress-energy tensor associated to the $\mathfrak{G}$-valued Yang-Mills curvature 2-form F_{ij}, where $\mathfrak{G}$ is the Lie-algebra of G, and $R_{ij}-\frac{1}{2}Rg_{ij}$ is the Einstein tensor computed with respect to the sought-for metric g_{ij}. If one considers static solutions, i.e., solutions depending only on r, and $G=SU(2)$, then (cf. [1]) we may write the metric as

$$ds^2=-T(r)^{-2}dt^2+A(r)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2), \qquad (1.1)$$

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