

# Jackson Integrals of Jordan–Pochhammer Type and Quantum Knizhnik–Zamolodchikov Equations

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**Abstract.** We show that the  $q$ -difference systems satisfied by Jackson integrals of Jordan–Pochhammer type give a class of the quantum Knizhnik–Zamolodchikov equation for  $U_q(\widehat{\mathfrak{sl}}_2)$  in the sense of Frenkel and Reshetikhin.

## 1. Introduction

One of the most interesting features of the Knizhnik–Zamolodchikov equation originated in conformal field theory is the relation between its connection matrix and the trigonometric solutions of the quantum Yang–Baxter equation [TK, K, D]. It is related to the fact that certain hypergeometric type integrals give solutions to the Knizhnik–Zamolodchikov equation [DJMM, Ma, Ch, SV], etc. This fact is also looked at from the viewpoint of the free field realization, e.g. [Ku, ATY]. Besides them, the structure of the hypergeometric type integrals had been studied, e.g. [A1, A2]. Recently it attracts attention to construct a  $q$ -analogue of these theories.

The Jackson integrals of Jordan–Pochhammer type are the simplest multivariable generalizations of Heine’s basic hypergeometric function which is a  $q$ -analogue of Gauss’ hypergeometric function. They satisfy a system of first order  $q$ -difference equations, whose connection problem was solved by Mimachi [Mi]. Recently Aomoto and others [AKM] showed that the connection matrix determined by Mimachi is related to the ABF-solution of the quantum Yang–Baxter equation [ABF]. On the other hand, Frenkel and Reshetikhin [FR] studied a  $q$ -analogue of the chiral vertex operators of the WZNW model, along the line of Tsuchiya and Kanie [TK]. In particular, they introduced a  $q$ -difference system called the quantum Knizhnik–Zamolodchikov equation, and discussed the relation of the connection matrix with elliptic solutions of the quantum Yang–Baxter equation. Then it seems possible to understand the result of [AKM] in the framework of Frenkel and Reshetikhin.

In this article, we shall explicitly give solutions to a certain class of the quantum Knizhnik–Zamolodchikov equation for  $U_q(\widehat{\mathfrak{sl}}_2)$  by Jackson integrals of Jordan–Pochhammer type. More precisely, we show that the  $q$ -difference system for the