Commun. Math. Phys. 151, 245–262 (1993)

Communications in Mathematical Physics

Topological Field Theory and Rational Curves

Paul S. Aspinwall[1] **and David R. Morrison**[2]

[1] University of Oxford, Department of Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK

[2] Department of Mathematics, Duke University, Durham, NC 27706, USA

Received October 24, 1991

Abstract. We analyze the quantum field theory corresponding to a string propagating on a Calabi–Yau threefold. This theory naturally leads to the consideration of Witten's topological non-linear σ-model and the structure of rational curves on the Calabi–Yau manifold. We study in detail the case of the world-sheet of the string being mapped to a multiple cover of an isolated rational curve and we show that a natural compactification of the moduli space of such a multiple cover leads to a formula in agreement with a conjecture by Candelas, de la Ossa, Green and Parkes.

1. Introduction

In its most fundamental form, string theory is normally considered as a loop of string propagating through space-time to sweep out a two-dimensional world-sheet. This map of the world-sheet into space-time allows one to "pull back" physics from the familiar space-time around us into a more simple two-dimensional quantum field theory. This model is usually cast in the form of the non-linear σ-model where the non-linearity of this field theory arises from curvature in the target space. Thus it is a simple matter to solve string theory in flat space-time, but more general curved target spaces can only be solved perturbatively assuming the curvature is small in some sense.

When one specializes to the case of requiring space-time supersymmetry one picks out a specific class of allowed target spaces in the above approximation, namely that the target space should have vanishing Ricci curvature and be a complex Kähler manifold. When building realistic models of physics one is naturally led in this situation to considering Calabi–Yau threefolds [1, 2].

This model might have been of interest only to superstring phenomenologists if it weren't for the fact that there is an alternative to this method of solution. That is, one can take an algebraic approach to string theory as a conformal field theory [3, 4]. This process does not involve the approximations required in the non-linear σ-model. The analogue of the Calabi–Yau condition in this case is that the