

The Logarithm of the Derivative Operator and Higher Spin Algebras of W_∞ Type*

Ioannis Bakas^{1**} Boris Khesin^{2****} and Elias Kiritsis^{3*****}

¹ Center for Theoretical Physics,*** Department of Physics and Astronomy, University of Maryland, College Park, MD 20742, USA

² Department of Mathematics, University of California, Berkeley, CA 94720, USA

³ Department of Physics,***** Theoretical Physics Group, Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA

Received October 22, 1991

Abstract. We use the notion of the logarithm of the derivative operator to describe W_∞ type algebras as central extensions of the algebra of differential operators. We also provide closed formulae for the truncations of $W_{1+\infty}$ to higher spin algebras with $s \geq M$, for all $M \geq 2$. The results are extended to matrix valued differential operators, introducing a logarithmic generalization of the Maurer–Cartan cocycle.

1. Introduction

The algebra of differential operators on the circle is becoming increasingly important in two dimensional physics, in particular in the theory of conformal models with extended (higher spin) symmetries, in the KP hierarchy of integrable differential equations and more recently in quantum gravity. Central extensions of this algebra provide a natural generalization of the Virasoro algebra which is generated by first order differential operators. It also contains the affine $U(1)$ current algebra generated by differential operators of zero order.

The algebra of differential operators on S^1 can be viewed as a deformation of the algebra of divergence-free (or Hamiltonian) vector fields on T^*S^1 . This relation can be easily understood by applying Leibniz's rule to find the commutator of two differential operators of order k and l ; indeed, the result is an operator of

* This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grants PHY-85-15857 and PHY-87-17155

** Present address: Dept. of Physics, Crete University, Heraclion, Greece.

*** e-mail: BAKAS@UMDHEP.bitnet, BAKAS@UMDHEP.UMD.edu, UMDHEP::BAKAS

**** Address after July 1, 1992: Dept. of Mathematics, Yale University, New Haven, CT06520, USA; e-mail: KHESIN@PASCAL.MATH.YALE.edu

***** Present address: Theory Division, CERN, Geneva, Switzerland

***** e-mail: KIRITSIS@LBL.bitnet, THEORY::KIRITSIS, KIRITSIS@THEORM.LBL.Gov, KIRITSIS@frulm62.bitnet