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String Theory and the Donaldson Polynomial

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Abstract. It is shown that the scattering of spacetime axions with fivebrane solitons of heterotic string theory at zero momentum is proportional to the Donaldson polynomial.

1. Introduction

A p-brane (i.e. an extended object with a $p+1$-dimensional worldvolume) naturally acts as a source of a $p+2$ form field strength F via the relation

$$\nabla^M F_{MN_1\cdots N_{p+1}} = q\Delta_{N_1\cdots N_{p+1}}\,, \tag{1}$$

where Δ is the p-brane volume-form times a transverse δ-function on the p-brane. In d dimensions they can therefore carry a charge

$$q = \int_{\Sigma^{d-p-2}} *F\,, \tag{2}$$

where the integral is over a $d-p-2$ dimensional hypersurface at spatial infinity. The dual charge

$$g = \int_{\Sigma^{p+2}} F \tag{3}$$

can be carried by a $d-p-4$ brane. A straightforward generalization [1] of Dirac's original argument implies that quantum mechanically the charges must obey a quantization condition of the form

$$qg = n\,, \tag{4}$$

just as for the special case of electric and magnetic charges in $d=4$. In particular, strings in ten dimensions are dual, in the Dirac sense, to fivebranes. Thus fivebranes are the magnetic monopoles of string theory.

In [2, 3] it was shown that heterotic string theory admits exact fivebrane soliton solutions. The core of the fivebrane consists of an ordinary Yang–Mills