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Generalized Classical BRST Cohomology and Reduction of Poisson Manifolds

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Abstract. In this paper, we formulate a generalization of the classical BRST construction which applies to the case of the reduction of a Poisson manifold by a submanifold. In the case of symplectic reduction, our procedure generalizes the usual classical BRST construction which only applies to symplectic reduction of a symplectic manifold by a coisotropic submanifold, i.e. the case of reducible "first class" constraints. In particular, our procedure yields a method to deal with "second-class" constraints. We construct the BRST complex and compute its cohomology. BRST cohomology vanishes for negative dimension and is isomorphic as a Poisson algebra to the algebra of smooth functions on the reduced Poisson manifold in zero dimension. We then show that in the general case of reduction of Poisson manifolds, BRST cohomology cannot be identified with the cohomology of vertical differential forms.

1. Introduction

Classical BRST cohomology has a long history in the physics literature, e.g. [1]. Although its origins are in the context of quantum field theory, it is now known that classical BRST cohomology is a cohomology theory that contains all of the information of the symplectic reduction of a symplectic manifold by a closed and embedded coisotropic submanifold [2, 3]. In the language of Dirac [4], this corresponds to symplectic reduction arising from (possibly reducible) "first class constraints." The classical BRST complex is constructed using only purely algebraic properties of the Poisson algebra of smooth functions on the original (unreduced) symplectic manifold and some of its ideals. Furthermore, since the classical BRST complex is a Poisson superalgebra and the differential a Poisson derivation, classical BRST cohomology inherits the structure of a Poisson

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