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Diagonalization of the XXZ Hamiltonian by Vertex Operators

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Abstract. We diagonalize the anti-ferroelectric XXZ-Hamiltonian directly in the thermodynamic limit, where the model becomes invariant under the action of $U_q(\widehat{\mathfrak{sl}}(2))$. Our method is based on the representation theory of quantum affine algebras, the related vertex operators and KZ equation, and thereby bypasses the usual process of starting from a finite lattice, taking the thermodynamic limit and filling the Dirac sea. From recent results on the algebraic structure of the corner transfer matrix of the model, we obtain the vacuum vector of the Hamiltonian. The rest of the eigenvectors are obtained by applying the vertex operators, which act as particle creation operators in the space of eigenvectors. We check the agreement of our results with those obtained using the Bethe Ansatz in a number of cases, and with others obtained in the scaling limit – the $su(2)$-invariant Thirring model.

0. Introduction

0.1. A Diagonalization Scheme. In this paper we give a new scheme for diagonalizing the 1-dimensional XXZ spin chain

$$H_{XXZ} = -\frac{1}{2}\sum_{k=-\infty}^{\infty} (\sigma_{k+1}^x\sigma_k^x + \sigma_{k+1}^y\sigma_k^y + \Delta\sigma_{k+1}^z\sigma_k^z)\,, \tag{0.1}$$

for $\Delta < -1$, directly in the thermodynamic limit, using the representation theory of the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}(2))$: we consider the infinite tensor product

$$W = \cdots \otimes \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \cdots\,, \tag{0.2}$$